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MATHEMATICAL MODEL OF OIL DISPLACEMENT BY WATER WITH THE USE OF POLYMER-DISPERSE SYSTEMS

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A mathematical model describing the process of displacement of oil by water with the use of polymer-disperse systems is proposed. The point of the technology consists of pumping sequentially water, a low-concentration polymer solution, and water with disperse particles. The porous medium is given in the form of two interpenetrating continua, one of which is associated with mobile phases and the other — with immobile phases. The corresponding expressions for dynamic porosity and the residual resistance factor have been obtained. In describing the mass transfer between the continua, an ideal model of a porous medium in the form of a bundle of capillaries of different radii and the pore- and particle-size distribution functions have been used.

Flooding of nonuniform oil pools is accompanied by early and rapidly progressing water encroachment on the products of oil wells. In so doing, at a late stage of exploiting oil fields, extensive washed zones characterized by a relatively high permeability are formed. In these zones the great bulk of pumped water is filtered without producing a significant effect on the production of weakly permeable intercollations. We have elaborated a new method for controlling this negative phenomenon consisting of increasing the filtration resistance of flooded zones under pool conditions by treating pools with polymer-disperse systems (PDS) [1]. The essence of the method consists of pumping sequentially water, a low-concentration polymeric solution (usually polyacrylamide), and water with disperse particles (particles of rocks) [2, 3].

The mechanism of the PDS action is determined by the following processes of interaction of the system's components with the pool and with one another [4]. The polymer macromolecules, moving in the pool, are adsorbed on the walls of the pore channels [5]. Note that the thin dense polymeric layer formed on the surface of rock grains does not lead to a significant change in the porosity and permeability. The ingress of hard particles into this medium is accompanied by the deposition of some of them on the walls of the pore channels due to their interaction with the free functional groups of the polymer fixed on the surface of the pores. The other part of them interacts with the polymer macromolecules in flowing liquid to form aggregates in the form of one or several particles with polymer macromolecules attached to them. The aggregates can be in a state of motion, and they can be deposited on the pore surface or mechanically held in the necks of the pore channels, considerably changing thereby the properties of the porous medium. The formation of aggregates deep in the pool on the walls of the pore channels and in free space causes a local increase in the filtration resistance due to the shrinkage and partial or complete blocking of individual pore channels, which in turn leads to a change of the directions of filtration flows and an increase in the degree of water flooding of the pool. Thus, when the above technology is used, changes in the dynamic porosity and permeability are observed, and in order to increase the coefficient of oil production, it is important to know in what place of

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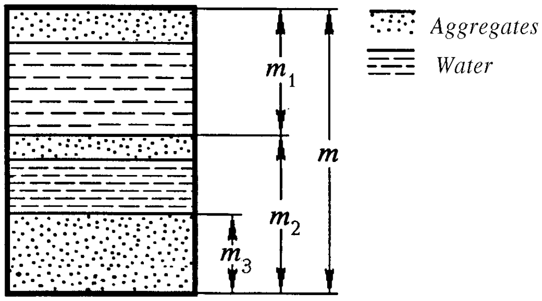

Fig. 1. Schematic arrangement of the pore space m: m_1, flowing liquid and polymer; m_2, immobile liquid and polymer; m_3, deposited clusters.

the pool this takes place and at what time and how the oil recovery will be affected depending on the particular conditions of displacement.

The aim of the present paper is to describe mathematically the process of oil displacement by water with the use of a PDS. To this end, we use the methods of continuous medium mechanics and the ideas expounded in [6, 7].

Mathematical Model. Let us assume that all reagents are carried by water alone and that the polymer and particles, depositing on the walls of the pore channels, decrease their flow area, and when stopped in the necks of the pores, they block the pore channels and immobilize the liquids contained in them.

We represent the porous medium in the form of two interpenetrating continua [8] and connect one of them with mobile liquids and the other one with immobile liquids. We distinguish four components: "oil," water, polymer, and particles. By particles we mean the rock particles proper and the aggregates formed as a result of the particle–polymer interaction.

Let $m_1 = m_1(x, y, z, t)$ be the part of the pore space occupied by mobile phases and $m_2 = m_2(x, y, z, t)$ — by immobile phases, and

$$m_1 + m_2 = m \,. \tag{1}$$

In the second continuum, we shall separately consider two volumes: (1) the part of the pore space in choked and deadlock pores; (2) the part of the pore space $m_3 = m_3(x, y, z, t)$ occupied by deposited aggregates (see Fig. 1). We neglect the intrinsic porosity, i.e., we assume that the deposited particles are closely packed and contain no liquid between them.

It may be assumed that at the initial time there are no blocked capillaries and the second continuum is given by bound water and residual oil.

We write the equation of conservation of the masses of phases and components in the large-scale approximation in the form

$$\frac{\partial}{\partial t}(m_1 S_{1i}) + \operatorname{div} \mathbf{U}_i = -\, q_i \,, \quad i = \text{o, w} \,; \tag{2}$$

$$\frac{\partial}{\partial t}(C_{1j} m_1 S_{1\text{w}}) + \operatorname{div}(C_{1j}\, \mathbf{U}_\text{w}) = -\, q_{1j} \,, \quad j = 1, 2 \,. \tag{3}$$

To the polymer there corresponds the index $j = 1$ and to the particles and aggregates — $j = 2$.

We write the equation of motion of phases in the form of the generalized Darcy law:

$$\mathbf{U}_i = -\frac{K_i}{\mu_i} \operatorname{grad}(P) \,, \quad i = \text{o, w} \,. \tag{4}$$

The dynamic viscosity of the phases depends on the concentration of impurities:

$$\mu_i = \mu_i\,(C_{11}, C_{12})\ .$$

The conservation equations for the second continuum are

$$\frac{\partial}{\partial t}\,(m_2 S_{2i}) = q_i\ , \quad i = \mathrm{o,\ w}\ ; \tag{5}$$

$$\frac{\partial}{\partial t}\,(C_{2j} m_2 S_{2\mathrm{w}}) = q_{2j}\ , \quad j = 1, 2\ . \tag{6}$$

The jth component’s mass removed from the mobile component will be composed of the masses of the component adsorbed on the porous skeleton, the component contained in the immobilized liquid, and the component that has interacted with another component.

Let us assume that at the initial time t in the unit volume of the porous medium in mobile water there are M disperse particles (aggregates) characterized by the particle-size distribution function $\psi(l, t)$. We assume that at the initial time all aggregates are represented by the disperse particles proper with the distribution function

$$\psi\,(l, 0) = \psi^0\,(l)\ . \tag{7}$$

Let the rate of enlargement of aggregates due to the addition of the polymer molecules alone be equal to λ. At the current instant of time the number of particles having the size from l to $l + \Delta l$ is equal to $M\psi(l, t)\Delta l$. In time Δt these particles will grow by the value of $\lambda\Delta t$. We assume that in time Δt their number will change by the value of $M\eta\Delta l\Delta t$ due to their deposition on the walls of the pore channels and mergence with other aggregates. We write the particle balance condition: $M\psi(l, t + \Delta t)\Delta l - M\psi(l, t)\Delta l = -M\psi(l + \Delta l, t)\,\lambda(l + \Delta l, t)\Delta t + M\psi(l, t)\lambda(l, t)\Delta t - M\eta\Delta l\Delta t$. Expanding the first term on the right side of the equality into a series, restricting ourselves to the terms of the first- order infinitesimal, and going to the limit at $\Delta t \to 0$, we obtain the so-called "continuity" equation [9] for the function ψ in the divergent form:

$$\frac{\partial\psi}{\partial t} + \frac{\partial(\lambda\psi)}{\partial l} + \eta = 0\ . \tag{8}$$

The aggregate enlargement rate is determined by the polymer–particle interaction kinetics and depends on the polymer and particle concentration and on the aggregate size. We assume that the interaction with the polymer does not affect the number of particles at each point of the pool (i.e., particles grow without merging) and take this dependence in the form

$$\lambda = \alpha\,(C_{11}^{*} - C_{11})\,C_{12}\,(l^{*} - l)\ , \tag{9}$$

where the asterisk marks certain limiting (critical) concentrations of the polymer and the particle size upon reaching which the interaction terminates.

The introduction into the enlargement rate of these critical values is due to the following experimental facts [3]. We noticed that enlargement of particles in the polymer was observed at small concentrations and ceased when the concentration exceeded a certain critical value. A particle cannot grow infinitely because of the limited number of sorption centers on it and by virtue of the fact that the added polymer macromolecules prevent the following macromolecules from getting into the sorption centers.

To describe the mass exchange between the two continua and the change in the filtration-volume characteristics of the pool caused by the process of aggregate deposition, we make use of the equation for the pore-size distribution function [8], which we also write in the divergent form

$$\frac{\partial \varphi}{\partial t} + \frac{\partial (u_{\mathrm{r}} \varphi)}{\partial r} + u_{\mathrm{n}} = 0 \,, \tag{10}$$

where u_{r} is determined by the dependence [6, 10]

$$u_{\mathrm{r}} = C_{12} S_{1\mathrm{w}} \left(\frac{2 u_{\mathrm{m}} D^2}{rL} \right)^{1/3} . \tag{11}$$

The value of u_{n} depends on both the size of the aggregates and the size of the pore channels and is determined proceeding from the model representation of the porous medium in the form of a bundle of capillaries of different radii, as was done in [8]. Taking into account the aggregate size distribution function, we can write

$$u_{\mathrm{n}} = - 6 \beta C_{12} \, S_{1\mathrm{w}} \, u_{\mathrm{m}} \varphi \frac{\int\limits_{d}^{\infty} \psi dl}{\int\limits_{0}^{\infty} \psi l^3 dl} . \tag{12}$$

The rate of deposition-mergence of particles can be given in the form of two terms: $\eta = \eta_{\mathrm{a}} + \eta_{\mathrm{c}}$. If the number of deposited particles of one size is proportional to their portion in the flow, then

$$\eta_{\mathrm{a}} = 2 u_{\mathrm{r}} \int\limits_{0}^{\infty} \psi l^3 dl \bigg/ \left(C_{12} S_{\mathrm{w}1} r \int\limits_{0}^{\infty} l^3 dl \right) . \tag{13}$$

The merging rate of particles η_{c} can be calculated in terms of the Smoluchowski coagulation theory [11].

In time Δt, the capillary radii will change, due to the deposition, by the value

$$\Delta r = u_{\mathrm{r}} \Delta t \,, \tag{14}$$

which will lead to a decrease in clearance. The new clearance (and, consequently, the porosity) will be

$$m_1 \, (t + \Delta t) = m_1 \int\limits_{0}^{\infty} \varphi \, (r - \Delta r)^2 \, dr \bigg/ \int\limits_{0}^{\infty} \varphi r^2 \, dr \,, \tag{15}$$

or, neglecting the $(\Delta r)^2$-containing term and taking into account (14), we obtain

$$m_1 \, (t + \Delta t) = m_1 \int\limits_{0}^{\infty} \varphi \, (r - 2 u_{\mathrm{r}} r \Delta t) \, dr \bigg/ \int\limits_{0}^{\infty} \varphi r^2 \, dr \,, \tag{16}$$

i.e., the clearance will change by the value

$$\Delta m_1 = -\,2m_1 \int_0^{\infty} \varphi u_r r \Delta t dr \Big/ \int_0^{\infty} \varphi r^2\, dr\,. \tag{17}$$

The quantity m_3 will change by the same value. Dividing (17) by Δt and letting Δt tend to zero, for m_3 we have

$$\frac{\partial m_3}{\partial t} = 2m_1 \int_0^{\infty} \varphi u_r r dr \Big/ \int_0^{\infty} \varphi r^2\, dr\,. \tag{18}$$

We estimate the change in the absolute permeability caused by the pore-space restructuring due to the deposition of aggregates and pore blocking by representing the permeability for a current instant of time $k_1(x, y, z, t)$ in the form of the product $k_1 = \bar{k}k^0$, where we determine the residual resistance factor $\bar{k}(x, y, z, t)$ making use of the model of parallel capillaries and the Poiseuille law

$$\bar{k} = \int_0^{\infty} r^4 \varphi dr \Big/ \int_0^{\infty} r^4 \varphi^0\, dr\,. \tag{19}$$

The mobile-immobile transition rate of water caused by the blocking of the pore channels can be calculated by the portion of the pore space passed into the second continuum:

$$q_{\mathrm{w}} = S_{1\mathrm{w}} m_1 \int_0^{\infty} u_{\mathrm{n}} r^2 dr \Big/ \int_0^{\infty} \varphi r^2 dr\,. \tag{20}$$

The mobile-immobile transition rate for oil is taken in the form

$$q_{\mathrm{o}} = (1 - S_{1\mathrm{w}})\, m_1 \int_0^{\infty} u_{\mathrm{n}} r^2 dr \Big/ \int_0^{\infty} \varphi r^2 dr\,. \tag{21}$$

The mobile-immobile transition rate of the polymer is equal to

$$q_{11} = C_{11}\, q_{\mathrm{w}} + q_{21}\,, \tag{22}$$

where $q_{21} = \partial a/\partial t$ and a is the adsorbed polymer mass determined by the sorption isotherm [8].

The mobile-immobile transition rate of particles is equal to

$$q_{12} = C_{12}\, q_{\mathrm{w}} + q_{22}\,, \tag{23}$$

where the aggregate deposition rate $q_{22} = \partial m_3/\partial t$.

In conclusion, note that a distinguishing feature of the proposed mathematical model is the presence in the general system of two additional equations, (8) and (10), describing the dynamics of changes in the pore- and particle-size distribution functions that are to be solved at each point of space in order to determine the changes in the structure of the porous medium.

NOTATION

m, initial porosity of the pool; S_{1i} and S_{2i}, saturation of the first and the second continuum of the pool with the *i*th phase; C_{1j} and C_{2j}, volume concentrations of the *j*th component in the first and the second continuum; C^{*}_{11}, limiting volume concentration of the polymer in the first continuum at which the aggregate growth terminates; $\mathbf{U}_i$, the ith phase filtration rate; $K_i = kf_i$, phase permeability; f_i, relative phase permeability; P, pressure; k^0, absolute permeability of the pool prior to the PDS action; k, permeability of the first medium; $\bar{k}$, residual resistance factor (coefficient characterizing the relative change in the first medium permeability); μ_i, dynamic viscosity of liquids; φ and ψ, pore and particle size distribution; λ, rate of particle growth as a result of the addition of polymer macromolecules; η, rate of deposition-merge of particles; η_{a} and η_{c}, rates of change in the number of particles in the flow caused by the deposition and merge of particles to form aggregates; u_{m}, mean velocity of motion of water in the pore channel; u_{r}, rate of change in the pore channel radii; u_{n}, rate of change in the number of pore channels of radius r; α, kinetic parameter; β, proportionality factor $(0 < \beta \le 1)$; q_i, rate of mobile-immobile transition of ith liquid; q_{1j}, rate of removal of jth component from the mobile continuum; q_{2j}, rate of mobile-immobile transition of jth component; r, pore channel radius; l, particle (aggregate) size; l^*, limiting particle (aggregate) size; t, time; L, characteristic length of pore channels; D, diffusion coefficient; $d = 2\gamma r$, diameter of the pore channel neck; γ, constant equal to the ratio of the neck radius to the pore channel radius. Subscripts: o, oil; w, water; $j = 1$ corresponds to polymer; $j = 2$ corresponds to particles and aggregates; a, deposition; c, mergence; m, medium; r, radius; n, number.

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